Topic Study Group 46

Mathematical Competitions and Other Challenging Activities

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ABSTRACT The joint focus of TSG-46 on mathematics competitions and other challenging activities is devised in recognition of the fact that all students benefit from studying mathematics through challenging activities but some students do not like to compete. This group gathered mathematicians, teachers, mathematics educators and mathematics education researchers and served as a stage for presentations and discussions related to the following themes: (i) Organizational formats for challenging students mathematically, (ii) Research on students' experiences with mathematically challenging activities, (iii) Characterizing and theorizing mathematical challenge, and (iv) Competition problems as impetus for mathematical research and discoveries.

Keywords: Mathematical challenge; Competitions; Characterization of tasks, Discovery.

1. Background and Agenda

The mathematics competitions movement emerged more than a century ago as a means to engage bright schoolchildren in mathematical activities that would be more challenging than activities traditionally included in regular mathematics curricula. There is overwhelming evidence that all students benefit from studying mathematics through challenging activities, though there are some students within every age cohort who require more mathematically advanced tasks than others do in order to be adequately challenged. In addition, it is well known that many students who enjoy feasible for them mathematical challenge, do not like to compete with other students. Hence, the joint focus of TSG-46 is on mathematics competitions and other challenging activities, within or beyond a mathematics classroom.

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The TSG-46 at ICME-14 built upon the work of the previous ICMI-initiated forums, such as the 16th ICMI Study "Mathematical challenge in and beyond the classroom", DG-16 at ICME-10, DG-19 at ICME-11, TG-34 at ICME-12 and TG-30 at ICME-13.

In spite of the essential work done in these forums, it still needs to be acknowledged that the mathematical challenge is an elusive notion. For instance, ICMI Study 16 suggests the following conceptualization of challenge:

> "…we will regard challenge as a question posed deliberately to entice its recipients to attempt its resolution while at the same time stretching their understanding and knowledge of some topic. Whether the question is a challenge depends on the background of the recipient; what may be a genuine puzzle for one person may be a mundane exercise or a matter of recall for another with more experience" (Barbeau & Taylor, 2009, p. 5).

This definition puts forward the expectations of the proposers of a challenge regarding actions of its (potential) recipients, but is rather silent about the recipient actual intentions and actions. Accordingly, the following queries are still open and require our attention as a community:

- Why and under which circumstances are our students inclined to accept or not the requests to invest intellectual effort in doing mathematical tasks with which their teachers attempt to challenge them?
- What are characteristics of the mathematical tasks that have a chance to be perceived by the students as engaging and feasibly challenging?
- What is the role of a competitive aspect of a mathematical challenge?
- How can tasks that are initially designed for the use in competitions be used in a regular classroom or in a teacher preparation workshop?
- What are the relationships between engaging students in challenging activities and fostering their creativity and mathematical habits of mind?

These and such queries have been at the heart of the discussions at TSG-46. As in the previous ICMEs, TSG-46 at ICME-14 gathered mathematicians, teachers, mathematics educators and mathematics education researchers and served as a stage for discussions related to the pivotal aspects of the group work: mathematical challenge, competitions, challenging activities connecting school mathematics and mathematics as a research area.

2. Submissions and Presentations

The initial Call for Papers attracted 16 high-quality submissions of different formats. Unfortunately, the changing the year and the format of the conference due to the corona-crises did not enable some of the authors to participate in the Congress. Eventually, 11 presentations have been delivered during three 90-minute sessions. Of note is that the group, though small, remained truly international and consisted

participants from 8 countries, as follows: Bulgaria, Colombia, Hungary, Israel (2), Portugal, China, Slovakia, USA (3).

The time of the sessions was distributed between whole-group discussions (45 minutes), two invited talks (IT), one long oral presentation (LO, 45 minutes including Q&A), and 7 short oral presentations and a poster (SO, 15 minutes including Q&A). See Tab. 1.

Tab. 1. The list of presentations

Paper and author(s):
Session 1
[1] What competitions can tell us about theories in mathematics education. ***Maria Falk de Losada*** (Colombia). (LO)
[2] How to identify multiple solution tasks for mathematical competitions. ***Ingrid Semanišinová****, Ľubomír Antoni, Stanislav Krajči, Daniala Vít'azková* (Slovakia). (SO)
[3] Challenging math tasks for teaching through problem solving approach. ***Hoyun Cho*** (USA). (SO)
Session 2
[4] Unravelling the construct of mathematical challenge based on conceptual characteristics of mathematical tasks. ***Roza Leikin*** (Israel). (IT)
[5] A challenge of deciding who is right and why. ***Reut Parasha****, Boris Koichu, and Michal Tabach* (Israel). (SO)
[6] Students' expected gains from a modeling competition. ***Elisabeth Roan*** *and Jenifer Czocher* (USA). (SO)
[7] Math trails: Opportunities to learn rich mathematics outside the classroom. ***Rosa Antonia Thomas Ferreira*** (Portugal). (SO)
Session 3
[8] Cutting a polygon: From mathematics competition problems to mathematical discovery. ***Kiril Bankov*** (Bulgaria). (IT)
[9] An introduction of Shanghai grade 11 mathematics competition. ***Yijie He*** *and Tianqi Lin* (China). (SO)
[10] POSA weekend-camps: A challenging mathematical environment for the highly gifted in Hungary. ***Eszter Bora*** (Hungary). (SO)
[11] Competitions promoting the mathematical science. ***Valorie Lynn Zonnefeld*** *and Ryan Glenn Zonnefeld* (USA). (Poster)

3. Thematic Overview of the Presentations

3.1. Organizational formats for challenging students mathematically

In line with a well-established tradition, TSG-46 at ICME-14 served as a stage for presenting unconventional formats of competitions and out-of-school activities aimed to challenge students mathematically. He and Lin[9] presented Shanghai Grade 11 Mathematics Competition, which encourages students to use various types of calculators, including graphical calculators. Examples of problems from this competition convincingly show that calculators can be used not only as technical scaffolds but as valuable tools for promoting students' mathematical creativity (e.g., by means of devising computational algorithms) and tools for developing conceptual understanding.

Zonnefeld and Zonnefeld[11] overviewed several types of thematic competitions, including a Math Bee, data-analytics competitions and March Madness competition, in which students design algorithms to select teams for a basketball tournament. Cho[3] described the "I Love Math Day" conducted annually on February 14 in an urban school in New Jersey. The "I Love Math Day" is a celebration that playfully recognizes student long-term mathematical problem-solving effort in small teams.

Bora[10] introduced "Pósa Weekend-Camps" — two-day-long mathematical workshops, in which more than 1500 Hungarian 6–11 grade students have taken part since 1988. A characteristic feature of these camps is that the problems chosen are organized in threads and form a rich network. In this way, students explore mathematics while experiencing a mix of discovery learning and guided learning.

3.2. Research on students' experiences with mathematically challenging activities

Students' experiences with competitions and other challenging activities were in the focus of two presentations[6,7]. In these presentation, organizational formats of challenging the students were described as contextual information, and empirical research was put forward.

Ferreira[7] analyzed the reactions of a class of 6th graders in Portugal on "Math Trail" consisting of a sequence of five stops along a predetermined path from a school to a city center and back to school. At the stops, the students were offered mathematical tasks combining intellectual, social and physical dimensions. The data for the tasks were gathered through direct observation of the environment at the stops. Overall, students' reactions on the entire experience were positive though some of the tasks appeared to be too cognitively demanding for them.

Roan and Czocher[6] presented a post-hoc analysis of pre- and post- survey data from two rounds of the "Challenge Using Differential Equations Modeling" (SCUDEM) competition for high-school and undergraduate students. In the SCUDEM competitions, teams of three work for a week on a modeling problem of their choice. At the competition site, teams are offered an additional modeling issue related to the problem they have handled, and then give a 10-minute presentation of their findings, which is followed by immediate feedback. The analysis revealed profound differences between expectations of students, researchers and problem designers in relation to the gains of the competition. For example, whereas researchers and designers expected the competition to be appreciated as an opportunity to engage in modeling and as a chance for recognition, the participants valued most the experience in modeling and teamwork skills.

3.3. Characterizing and theorizing mathematical challenge

In her invited talk, Leikin[4] considered mathematical challenge embedded in a task as a complex function of the task characteristics, didactical settings in which the task is approached, classroom socio-mathematical norms and mathematical potential of the students who cope with the task. In particular, she focused on such task characteristics as conceptual density, the task's openness, and the complexity of mathematical

concepts, mathematical connections and logical relationships required for solving the task.

Semanišinová et al.[2] reported a novel method of analyzing competition tasks by means of "formal concept analysis". The method was illustrated by its application to two multiple-solution problems from the Slovak correspondence mathematical competition. The fine-grained analysis of students' solutions revealed that while expert solution spaces were wide enough for both problems, the collective solution spaces varied. In conclusion, the scholars advocated for the choice of competition tasks that would be less dependent on the knowledge of particular mathematical concepts or specific solution methods.

Parasha et al.[5] introduced a notion of "dialogical challenge", that is, a mathematical challenge associated with collective argumentative activity towards deciding which of contradictory solutions to a problem is right and why. By means of three examples, they argued that a dialogical challenge is two-dimensional: the first dimension is related to understanding and validating the solutions, and the second one — to inventing an argument that would be convincing to the peer students.

Finally, a long oral presentation of Falk de Losada[1] was devoted to theoretical analysis of competition problems by means of the conceptual apparatus suggested by Duval's theory of semiotic registers. She provided illuminating examples of the competition problems that can be solved by changing a semiotic register. Based on these examples, Falk de Losada argued that mathematical thinking is more than representations and treatments, and mathematics is more than a language to be learned, but rather an elastic medium that supports the search of novel and unanticipated connections.

3.4. From competition problems to mathematics discoveries

The invited talk of Bankov[8] elaborated on mathematics competition problems as intellectual products, which can serve as impetus for mathematical discoveries. A classic competition context of "cutting a polygon" was unfolded in the lecture. Inspired by a beautiful problem from the 1968 Saint Petersburg Mathematics Olympiad, Bankov considered a series of follow-up questions and problems, some of which are within the reach of a bright school student, some — require profound mathematical knowledge, and some are still open.

Interestingly, one of the open questions mentioned in the lecture deserved special attention at the summarizing discussion of the group when one of the participants (Sergei Dorichenko) offered an idea for how the question can be answered. The vivid discussion of the Dorichenko idea, which has been formulated on the spot, expressed greatly the creative spirit of TSG-46.

References

E. J. Barbeau and P. J. Taylor (eds.) (2009). *Challenging Mathematics in and beyond the Classroom*, Study Volume of ICMI Study 16. New York, NY: Springer.